Spring 2015 | **Quiz 1** | **Date: February 21**
K.Yaghi | **Math 201- Section X** | **Duration: 1 hour**

Problem 1 *(answer on page* 1 *of the booklet)*
Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence. (7 *pts each*)

a) $a_n = (1 + \frac{(-1)^n}{n^2})^n$ b) $b_n = \left(\sqrt[n]{n} - 1\right) \ln n$ c) $c_n = \frac{1}{(n^{\frac{1}{\ln n}}+1)^{1/n}}$

Problem 2 (*answer on pages* 2 & 3 *of the booklet*)
Which of the following series converge, and which diverge? When possible find the sum of the series. (7 *pts each*)

a) $\sum_{n=1}^{\infty} \frac{3^n}{8^{n-1}} + \frac{3}{n(n+3)}$ b) $\sum_{n=2}^{\infty} \sin(\frac{1}{\ln^2 n})$ c) $\sum_{n=3}^{\infty} (-1)^n \cos(\frac{1}{n+1})$ d) $\sum_{n=2}^{\infty} \frac{n \ln n}{(n+1)^4}$ e) $\sum_{n=2}^{\infty} \frac{\sqrt{2+\frac{1}{n^2}} - \sqrt{2}}{\sqrt{2+\frac{1}{n^2}}}$

Problem 3 (*answer on page* 4 *of the booklet*)
Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2 4^n} (x+7)^n$$

For what values of x does the series converge absolutely? Conditionally? (20 *pts*)

Problem 4 (*answer on page*s 5, 6 *and the last page of the booklet*)

a) (5 *pts*) Prove that

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \qquad |x| < 1 \qquad (1)$$

b) (7 *pts*) Find the taylor polynomials $p2(x)$ and $p3(x)$ generated by $\arctan x$ about the point $x = 0$. Then use the alternating series estimation theorem to estimate the error resulting from the approximation

$$\ln(0.7) \approx p3(?)$$

Does $p3(?)$ tend to be too small or too large?

c) (7 *pts*) Use taylor's theorem to prove that

$$\ln(1+x) < x \qquad 0 < x < 1$$

(Hint: you have to prove that the error resulting from the approximation $\ln(1+x) \approx x$ is strictly negative for $0 < x < 1$)

d) (5 *pts*) Decide if $\sum_{n=2}^{\infty} n \left(\frac{1}{n} - \ln(1+\frac{1}{n})\right)^{1.2}$ converge or diverge. Justify

e) (4 *pts*) What about $\sum_{n=2}^{\infty} n \ln(1+\frac{1}{n})$?